

## Middle School Mathematics - Differentiation Strategies that Work

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## Why do we study mathematics in school?

"Because my teacher could get sued if we don't. That's what she said. Any subject we don't know...wham! She gets sued.

And she's already poor."

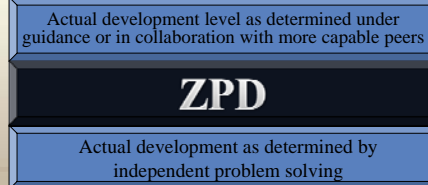
Cory, Age 9

## What is Differentiation?

- Curriculum differentiation is a process used to **maximize student learning** by **improving the match** between a student's individual needs and the curriculum.
- A general term used to describe the **range of strategies**, which are used to ensure children's needs are met.
- Curriculum differentiation is a broad term referring to the need to tailor teaching environments and practices to create appropriately **different learning experiences for different students**.
- Adapting the curriculum to meet the unique needs of learners by making **modifications in complexity, depth, and pacing**.

## Differentiation...

Allows each student to work in his or her zone of proximal development  
(state of moderate challenge)



## ZPD

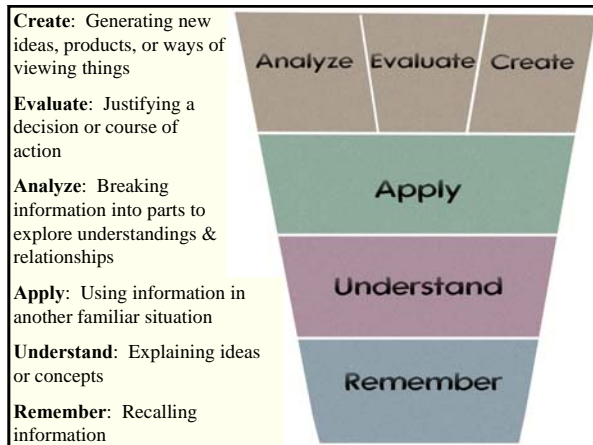
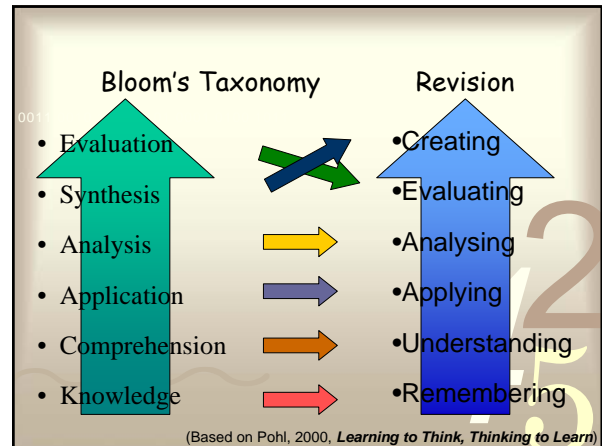
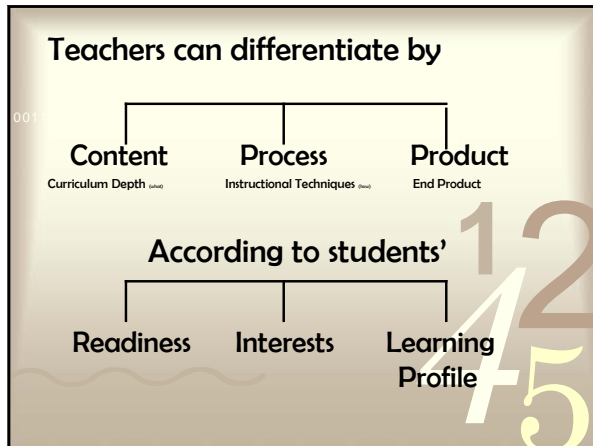
- Too Easy
  - Already knows
  - Gets it quickly
  - No effort needed
- Moderate Challenge
  - Know something
  - Have to think
  - Must persist
  - Effort leads to success
- Too Hard
  - Don't know where to start
  - Missing skills
  - Can't solve
  - Makes no sense



Based on C. Tomlinson, 2004

## Why Differentiate?

- Pressure to standardize learning
- Students differ in abilities, interests, and learning styles
- Learning more enjoyable when choices are available
- Enjoyment and engagement contribute to higher achievement
- And, of course, the state says you must!



- ### Differentiation Strategies
- 001
- Curriculum Compacting
  - Independent Projects
  - Tiered Assignments
  - Flexible Grouping
  - Learning or Interest Centers
  - Varying Questions
  - Mentorships
  - Learning Contracts

### To differentiate you must...

001

- Know your learning goals
- Know the ability range of your students

THE MOMENT OF INERTIA FOR A SOLID SPHERE IS  $\frac{2}{5} MR^2$

MEANWHILE, THE MOMENT OF INERTIA FOR US IS...

THE FIRST NANOSECOND OF SUMMER VACATION, OF COURSE.

I WISH YOU BOYS WOULD GET OFF THAT SOFA!

THE MOMENT OF INERTIA FOR A HOLLOW SPHERE IS  $\frac{2}{3} MR^2$

THE MOMENT OF INERTIA FOR A SOLID CYLINDER IS  $\frac{1}{2} MR^2$

- ### The Value of Assessment or ...
- 001
- You can't figure out what to teach 'em if you don't know 'em!**
- ❑ Interest Inventories
  - ❑ Learning Profile Inventories
  - ❑ Preassessment Options - Ensure the Mastery of Basic Skills!

## Ensure the Mastery of Basic Skills:

### Mastery

Recognition of situation requiring repeated addition, uses multiplication to shorten solution process

Uses variety of basketball passes depending on best strategy for the moment

Can explain how the role of a number changes based on its placement in the equation

### Not Mastery

Can automatically recite multiplication facts

Primarily uses the bounce pass in basketball regardless of its potential effectiveness

Can recite the definition of a mathematical property.

Adapted from Wasmith, 2006

## Preassessment Options

- Textbook Pretest
- Student/Teacher Conference - as short as a 5 minute talk
- K-N-W Chart - What do I Know, Need to know & Want to know
- Journal - Write what you know about...
- List - If I say ...
  - What does it make you think of?
- Product - Draw a bar graph...
  - Use the graphing calculator to plot...
- Concept Map...
- Five Hardest
- Exit Cards

## EXIT CARDS

- "Tickets To Leave" or Nugget
- Quick assessment tool
- Students respond to teacher prompt
- Teacher uses responses to determine readiness and/or interests

Students who are struggling with the

Students who have some, but not a complete, understanding

Students who have mastered the concept

Readiness Groups

## Sample Prompts

- What one thing will you remember most about today's lesson.
- List 3 things you learned today.
- List 2 examples of....
- What questions do you have about...
- Explain the difference between...
- What area gave you the most difficulty today?
- Something I still don't understand it...

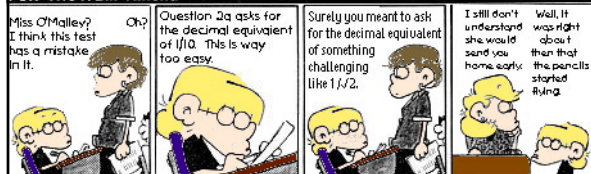
## Questions to ask as you plan...

Will what I have planned...

- ☐ Meet all of the student's needs?
- ☐ Be necessary for all students?
- ☐ Meet the needs of students who learn quickly?

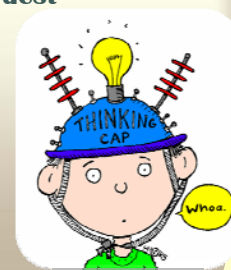
How will I know that students have mastered the material?

**FOX TROT/Bill Amend**



## Challenge through Choice

- ☐ Questioning
- ☐ Compacting/Five Hardest
- ☐ Anchor Activities
- ☐ R.A.F.T.
- ☐ Think, Pair, Share
- ☐ Think Tac Toe
- ☐ Tiered Activity



## Questioning in the Classroom

Haynes	1935	70% recall	17% thinking
Gall	1970	60% recall	20% thinking
Kroll	1980	29 % recall 47% routine	23% thinking
Kerry	1989	54% routine 42% recall	4% thinking
Wragg	1993	57% routine 35% recall	8% thinking
Lincoln	1996	60% recall 20% routine	20% thinking

## Ask Open Ended Questions

- Take away the question.
  - Three ducks and 2 ducklings weigh 32 kg. Four ducks and 3 ducklings weight 44 kg.
- Here is the answer, what is the question.
  - The answer is 27
  - The answer is  $\geq 32$
- Give an example of an event that has a probability of 0. Provide proof.

## Wait Time

- Provide time for reflection
- Students may be resistant to “having to think”
- Wait Time
  - Averages one second or less.
  - Students whom teachers perceive as slow or poor learners are given less wait-time than those teachers view as more capable.
  - Increase in wait-time over three seconds has a positive effect on the number of higher cognitive questions asked by teachers.
  - Got the answer? Give me a hint...

## Curriculum Compacting

- Used to modify and/or streamline the regular curriculum to eliminate repetition of previously mastered material, upgrade the challenge level of the regular curriculum, and provide time for enrichment and/or acceleration activities.

## Compacting Steps

1. What do you want them to know?
2. What do they know?
3. Offer enrichment or acceleration activities to those who already know it.
4. Keep records for accountability.



<http://www.gifted.uconn.edu/siegle/CurriculumCompacting/INDEX.HTM>

## Five Hardest (Short Cuts)

- Teacher or student selects the 5 (or other number) most difficult problems on the page
- Do with 80% accuracy and...
- NEATLY!
- Buy self out of the remainder of the problems on the page

## Learning Contracts

- An agreement between student and teacher
  - student directed
- Instead of, not in addition to...
- Streamline delivery or eliminate mastered content

What	How	When	Proof to self	Proof to audience

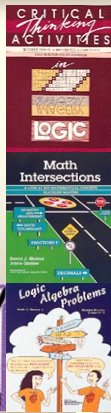
## Anchor Activities



- Self-paced, purposeful, content-driven activities that students can work on independently during a unit or grading period
- Meaningful ongoing activities related to the curriculum
  - A list of activities that a student can do at any time
  - A long-term project
  - An activity center/learning station located in the room
- These activities must be worthy of a student's time and appropriate to their learning needs

## A few possibilities:

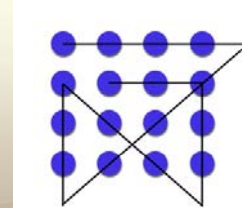
- Logic problems
- Pre-algebra thinking activities
- Open-ended math problems
- Figural Analogies
- Spatial Reasoning problems
- Visual Analogies
- Unit projects



## Spatial Reasoning Puzzles

### CIRCLES:

Using six contiguous straight lines, connect all of the sixteen circles shown below.



## Management Suggestions

- Explain the activity and the procedures with the whole class
- Make expectations clear – develop ground rules for:
  - Behavior
  - Performance
- Use tasks that require time and thinking – this is not an extension of the “seat-work” concept
- Provide clear instructions, materials, responsibilities, check points, and expectations (rubrics)



## RAFT

Role: Who are you?	Audience: To whom is this written?	Format: What form will it take?	Topic: What is the topic?
One parallel line	The other parallel line	Λ (love) rejection letter	Why we just don't connect?
Λ vertical line	Λ horizontal line	Advice column	I how not to be so flat?
Origin (0, 0)	Any other coordinate	Riddle	Why I am so special?
Linear Equation	Itself	Journal	Why do I always take the straight and narrow path?
Standard form (Ax + By = C)	Slope-Intercept Form (y = mx + b)	A Friendly letter	We are just alike.

Role	Audience	Format	Topic
Exponent	Jury	Instructions	Laws of Exponents
Acute Triangle	Obtuse Triangle	Dear John Letter	Our Differences
Percent	Student	How-To Guide	Mental ways to calculate percent
Prime Number	Rational Numbers	Club Membership Form	How to Join My Club
Parts of a Graph	TV Audience	Script	Which of Us Is Most Important?
Plus Sign	Multiplication Sign	Romantic Card	Why We Go Together

<http://wvde.state.wv.us/strategybank/RAFT.html>


Role	Audience	Format	Topic
Zero	Whole numbers	Campaign speech	Importance of the number 0
Scale factor	Architect	Directions for a blueprint	Scale drawings
Percent	Student	Tip sheet	Mental ways to calculate percents
Repeating decimal	Customers	Petition	Proof/check for set membership
Prime number	Rational numbers	Instructions	Rules for divisibility
Parts of a graph	TV audience	Script	How to read a graph
Exponent	Jury	Instructions to the jury	Laws of exponents
One	Whole numbers	Advice column	Perfect, abundant, deficient, amicable numbers
Variable	Equations	Letter	Role of variables
Container	Self	Diary	Comparing volume measurements
Acute triangle	Obtuse triangle	Letter	Explain differences of triangles
Function	Relations	Article	Argue the importance of functions

<http://www.tantasqua.org/Superintendent/Profdevelopment/etraff.html>

## Think, Pair, Share

- Think about the problem (5 minutes or less)
- Pair up - Share thoughts with a classmate
- Pair up pairs - Share your thoughts

And the problem is....  
Create as many problems as you can based on the figure below:










## Think-Tac-Toe

- Blooms Taxonomy
- Multiple Intelligences
- By Readiness
- Choice

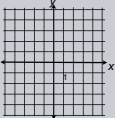
• A simple way to give students choices.

• Activities should be structured so that students must grapple with the key ideas and use the key skills central to the topic or area of study

	Free Choice	
		
	Free Choice	

## Think-Tac-Toe or Tic-Tac-Toe

Complete question # .... on page .... in your text.	Argue pro or con: The best way to add mixed numbers is to make them into equivalent improper fractions.	Think of a situation where you would add fractions in your everyday life.
Make up a jingle that would help someone remember the steps for subtracting mixed numbers.	Someone asks you why you have to get a common denominator when you add and subtract fractions but not when you multiply. What would you say?	Create a subtraction of fractions question where the difference is $\frac{3}{5}$ . • Neither denominator you use can be 5. • Describe your strategy.
Replace the blanks with the digits 1, 2, 3, 4, 5, and 6 and add these fractions: $\frac{\square}{\square} + \frac{\square}{\square} + \frac{\square}{\square}$	Draw a picture to show how to add $\frac{3}{5}$ and $\frac{4}{6}$ .	Find or create three fraction "word problems". Solve them and show your work.

Write the equation of a line that is <i>parallel</i> to this line.	Find the $x$ and $y$ intercepts.	Write the linear equation in <i>slope-intercept</i> form.
Write the linear equation in <i>point-slope</i> form.	What is the slope of the line?	Find 2 points on the line ( <i>not</i> intercepts).
Draw the graph of the line. 	Write the linear equation in <i>standard</i> form.	Write the equation of a line that is <i>perpendicular</i> to this line.


<p><b>Compare and Contrast</b></p> <p>Study the steps of factoring <math>ax^2+bx+c</math> by opening the PowerPoint presentation at <a href="#">this site</a>. Then, Print out and complete the <b>Venn Diagram</b> to compare and contrast factoring when a is one and when a is not one.</p>	<p><b>GCF Practice</b></p> <p>Visit this <a href="#">website</a> to get interactive practice factoring the GCF out of an algebraic expression.</p>	<p><b>Read-a-Picture</b></p> <p>Visit this <a href="#">website</a>. After studying the pictures, complete the questions.</p>
<p><b>Kinesthetic Testing Tactics</b></p> <p>Use these <a href="#">interactive algebra tiles</a> to confirm your factoring results. (You have to have java to use this tool)</p>	<p><b>Timeline</b></p> <p>Choose eight important events in the life of one of the mathematicians on your handout. Use the <a href="#">online timeline generator</a> to create a timeline of the events.</p>	<p><b>Walk a Mile</b></p> <p>What was a day in the life of the mathematician (that you chose for your timeline) like? <a href="#">Follow the directions</a> to "walk a mile" in their footsteps.</p>
<p><b>Jeopardy</b></p> <p>Click here to play <a href="#">Factoring Jeopardy</a>.</p>	<p><b>Brochure</b></p> <p>Make a brochure about factoring using the <a href="#">online brochure maker</a> or by following the <a href="#">directions</a>.</p>	<p><b>Make Your Case</b></p> <p>Download the <a href="#">template</a> to make a CD case for a CD pertaining to factoring. All the song names should be related to factoring.</p>

http://www.bedfordk12n.com/education/components/sci/apbook/default.php?action=detail&id=1153&pagecat=159&PHPESSID=48c618d22cda164943649248c7244 created by Kellen Anderson

## Tiered Activities

Tiered Instruction features:

- ✓ Whole group introduction and initial instruction
- ✓ Identification of developmental differences
- ✓ Increase or Decrease the:
  - ✓ Abstraction
  - ✓ Extent of Support
  - ✓ Sophistication
  - ✓ Complexity of goals, resources, activities & products



## What constitutes a tiered activity?

- A focus on a key concept – parallel tasks
- Adjust to students' achievement levels
- Adjust number of steps to the students' productivity levels
- Students working with appropriately challenging tasks
- Result = Respectable work for everyone
- Students understand why they are all not doing the same thing.

Mathematical Processes Algebraic Relationships	Algebraic Relationships Geometry	Algebraic Relationships Geometry
<b>Task to be Accomplished</b>	<b>Task to be Accomplished</b>	<b>Task to be Accomplished</b>
The cost of bringing in CCR for the concert is \$20,000.00. How many tickets must be sold for \$10.00 each to make a profit of \$10,000.00 for Harbor House?	Develop a plan for seating in the gymnasium. Remember to leave room for a stage, aisles that meet fire codes, and paths. You will need to determine which chairs will be used. How many will fit? Use a diagram to represent the scale so that you can estimate how many chairs you can fit. Complete your results in chart form.	With such a huge parking place at CMS, parking may be a problem. What is the maximum number of vehicles that can be parked at CMS? Estimate the number of vehicles that can be parked for the number of tickets that must be sold. You will have to talk to members of Groups 1 and 2 to find out many tickets need to be sold. Also, you will have to assume that each vehicle will have an average of 2 people in it. Coordinate the findings from all groups into a 10 minute presentation of a proposal.

TIME TO ROCK CONCERT!!!

## Vacation Time!

Calculate approximate cost of gas

<b>Prompt One</b> Given the cost of gas and mpg of car	<b>Prompt Two</b> Given mpg of car	<b>Prompt Three</b> Asked to approximate cost and justify answer
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Family plans to average 50 miles per hour and travel 6 hours per day stopping twice to eat for an hour each time.

How long will it take to get to their destination? How many nights? Cost of hotel is about \$80 per night – calculate hotel cost for trip to destination.	How long will it take? How many nights will they need to spend in hotels? Calculate hotel cost for trip to destination.	How long? How many nights? Find hotels that include breakfast. Calculate hotel costs for each stay.
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## Beginning Probability

**Task 1:** It's early Monday morning and your mother has laid out the following clothing items for you to choose from: a red shirt, a blue shirt, a white shirt, blue jeans, and khaki pants. How many different outfits can you make with the clothes your mother has provided?

**Task 2:** You are making cupcakes for a class celebration. Your classmates have indicated that they would like a choice of different cupcakes. You have: chocolate and yellow cake batter; strawberry, white, and caramel icing; and green and blue sprinkles. How many different types of cupcakes can you offer your classmates?

**Task 3:** You are trying to determine your schedule for next year at Leonard Middle School. First period, you can take art, chorus, or band. Second period, you can take technology or creative writing or be an office assistant. Third period, you can take a foreign language: German, Spanish, French, or Latin. Figure out how many different schedules are possible based on these options.

## Mean, Median and Mode

- Compute the mean, median, and mode for:

– Task 1:

2 4 4 6 4 2

– Task 2:

3 7 8 7 8 2

– Task 3:

4.2 5 6 8.1 9.3 2.7

## Tiered Assessments

Jakarta International School

<http://challengebychoice.wordpress.com/examples-of-tiered-math-assessments/>

Write a variable word phrase for:

Tier 1: The number of eggs in  $m$  dozen

Tier 2: 5 less than the quotient of 10 and the product of 2 and a number

Tier 3: Hot water flows at 8.7 liters per minute. Two minutes later you also turn on the cold water, which flows at 13.2 liters per minute. Let  $x$  be the number of minutes since you turned on the cold water. Write an expression for the number of liters the hot water has delivered.

1. Should every student do it?

Yes  No

2. Would every student want to do it?

3. Could every student do it?

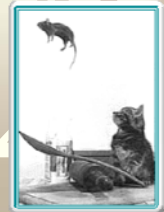
## Remember:

Start small

Make friends and share

Your mantra:

“Different, not more”



CLOSE TO HOME

BY JOHN McPHERSON



THE SECRET BEHIND OLD FAITHFUL.

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